CLASA A V-a
BAREM

(1) $A = (2024-2023)+\ldots+(2-1)$ (2p)

$= 1+\ldots+1$ (2p) $= 1012$ (3p)

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(2) $H = 2^{2024}(1+2)\cdot 5^{2025}(1+5)$ (2p)

$= 18\cdot 2^{2024}\cdot 5^{2024}\cdot 5$ (2p)

$= 90\cdot 10^{2024} = 900\ldots00$ (2p)

$S(N) = 9$ (1p)

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(3) $a = (5+5^2+5^3+5^4)+\ldots$ (2p)

$= 5(1+5+5^2+5^3)+\ldots$ (2p)

$= (1+5+5^2+5^3)(5+5^5+\ldots)$ (2p)

$= 2\cdot 78\,(5+5^5+\ldots)$ (1p)

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(4) $a \leq 1$ (1p) $b \leq 1$ (1p)

$\Rightarrow a^2+b^{25} \in \{0,1,2\}$ (1p)

$\Rightarrow 20^c + 25^d \in \{8627, 8626, 8625\}$ (1p)

$c \leq 3$ (1p)

$c=3 \Rightarrow d=2 \Rightarrow a=1, b=1$ (2p)

CLASA A VI A

BAREM

(1) $x = \frac{1}{5}(180^\circ - x)$ (2p) $\Rightarrow x = 30^\circ$ (2p)

$90^\circ - x = 60^\circ$ (3p)

(2) $(a,b)+1)([a,b]+1) = 2 \cdot 5$ (2p)

$(a,b)+1 = 2$; $[a,b]+1 = 5$ (2p)

$(a,b) = 1$; $[a,b] = 4$ (2p)

$a = 1 \cdot b = 4$ (1p)

(3) PRA că niciunul nu se divide cu 97 (1p)

Cele 97 elemente dau resturile 1, 2, ..., 96 (2p)

Conform principiului cutiei, există $i > j$ a.î. 11...1 (i cifre) și 11...1 (j cifre) dau același rest la împ. cu 97. (2p)

$\underbrace{11\ldots1}_{i} - \underbrace{11\ldots1}_{j} = \underbrace{11\ldots1}_{i-j}\underbrace{00\ldots0}_{j} \vdots 97$ (1p)

$\Rightarrow \underbrace{11\ldots1}_{i-j} \vdots 97$ (1p)

(4) $|A| = 676$, $|B| = 406$, $|A \cap B| = 136$ (2p)

$|A \cup B| = 676 + 406 - 136 = 946$ (1p)

b) $k \in A \cup B \Rightarrow 2025 - k \in A \cup B$ (1p)

$A \cup B = \{a_1, a_2, \ldots, a_{946}\} = \{2025 - a_1, \ldots, 2025 - a_{946}\}$ (1p)

$a_1 + \ldots + a_{946} = (2025 - a_1) + \ldots + (2025 - a_{946})$ (1p)

$a_1 + \ldots + a_{946} = \frac{1}{2} \cdot 2025 \cdot 946 = 957.825$ (1p)

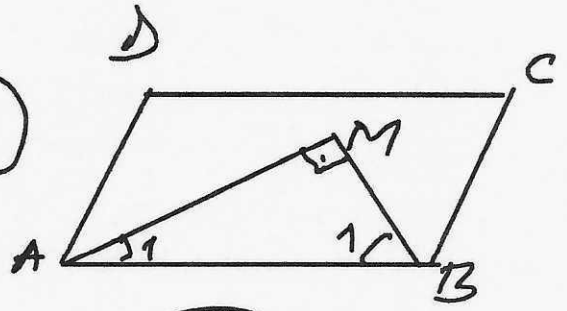
CLASA A VII-A

BAREM.

(1) $\hat{A}_1 + \hat{B}_1 = \frac{1}{2}\hat{A} + \frac{1}{2}\hat{B}$ (2p)

$= \frac{1}{2}(\hat{A} + \hat{B}) = 90^\circ$ (2p)

$\hat{M} = 180^\circ - (\hat{A}_1 + \hat{B}_1) = 90^\circ$ (3p)

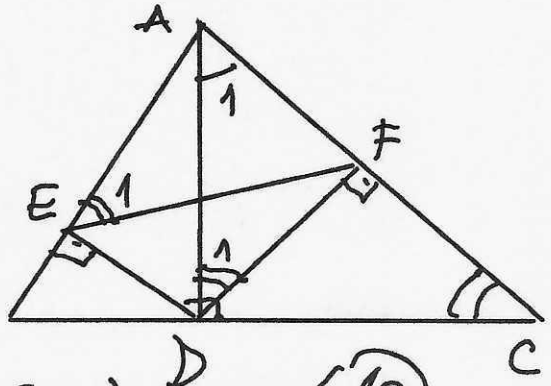


(2) $AEDF$ - inscr (2p)

$\hat{E}_1 = \hat{D}_1$ (2p)

$\Delta ADC: \hat{D}_1 = 90^\circ - \hat{A}_1$ (1p)

$= \hat{C}$ (1p) $\Rightarrow \hat{E}_1 = \hat{C} \Rightarrow BEFC$ inscr (1p)



(3) $S = \sum_{k=1}^{2025}\left[\frac{k^2}{k} + \frac{k-1}{k}\right]$ (2p) $= \sum_{k=1}^{2025}\left[k + \frac{k-1}{k}\right]$ (1p)

$= \sum_{k=1}^{2025} k$ (2p) $= \frac{2025 \cdot 2026}{2}$ (1p) $= 2.051.325$ (1p)

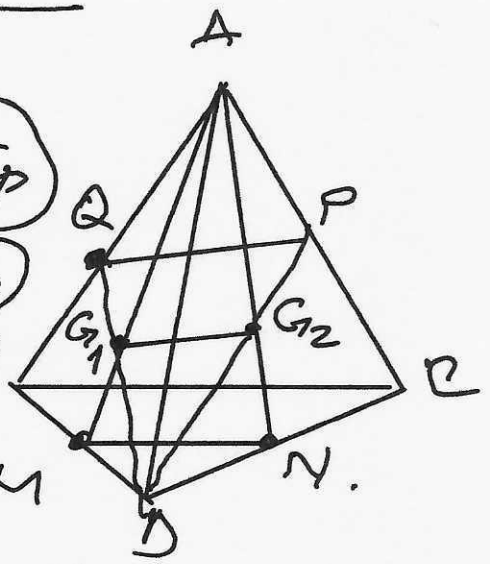
(4) $a\sqrt{2} + b\sqrt{3} = \frac{(a\sqrt{2} + b\sqrt{3})^4}{(a\sqrt{2} + b\sqrt{3})^3}$ (2p)

$= \frac{88\sqrt{6} + 217}{34\sqrt{2} + 27\sqrt{3}}$ (1p), amplif. conj (1p)

$= 2\sqrt{2} + \sqrt{3}$ (2p) $\Rightarrow a = 2, b = 1$ (1p)

CLASA A VIII-A
BAREM

(1) In $\triangle AMN$: $G_1G_2 \parallel MN$ (1,5p)
$MN \subset (BCD) \Rightarrow G_1G_2 \parallel (BCD)$ (2p)
In $\triangle DPQ$: $G_1G_2 \parallel PQ$ (1,5p)
$PQ \subset (ABC) \Rightarrow G_1G_2 \parallel (ABC)$ (2p)



(2) $S = \frac{\sqrt{5}-\sqrt{1}}{(\sqrt{5}-\sqrt{1})(\sqrt{5}+\sqrt{1})} + \dots$ (2p)
$= \frac{1}{4}(\sqrt{5}-\sqrt{1}) + \dots$ (2p) $= \frac{1}{4}(\sqrt{2025}-1)$ (2p)
$= 11$ (1p)

(3) $x + \frac{1}{x} = 3$. (2p) $x^2 + \frac{1}{x^2} \in \mathbb{Q}$ (2p)
$x^3 + \frac{1}{x^3} \in \mathbb{Q}$ (2p) Concluzie (1p)

(4) $(x-1)^2 + (y+1)^2 \le 4$ (1p)
$(x-1)^2 \le 4 \Rightarrow -2 \le x-1 \le 2$ (1p)
$-1 \le x \le 3$ (1p)
$(y+1)^2 \le 4 \Rightarrow -2 \le y+1 \le 2$ (1p)
$-3 \le y \le 1$ (1p)
$x - y \le 3 - (-3) = 6$ (1p)
$x - y = 6 \Rightarrow x = 3, y = -3$ imp (1p)

CLASA IX-A

BAREM

1 $(k-1)^2 < k^2 - k + 1 < k^2$, $k \geq 2$. (2p)

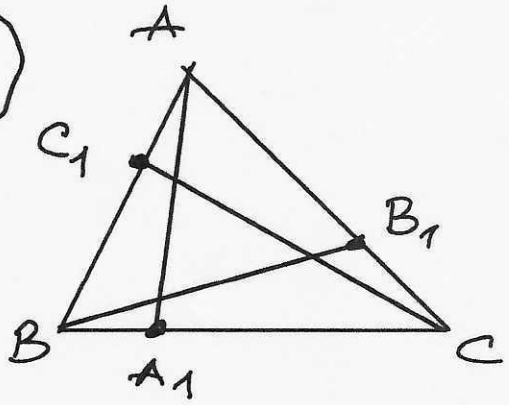
$\Rightarrow \left[\sqrt{k^2-k+1}\right] = k-1$, $k \geq 2$. (2p)

$S = 1 + \sum_{k=2}^{n} (k-1)$ (2p) $= 1 + \frac{n(n-1)}{2}$ (1p)

2 $\overrightarrow{AA_1} = (1-k)\overrightarrow{AB} + k\overrightarrow{AC}$ etc (2p)

$\sum \overrightarrow{AA_1} = \sum \{(1-k)\overrightarrow{AB} + k\overrightarrow{AC}\}$ (2p)

$= \vec{0}$ (2p) Concluzie (1p)

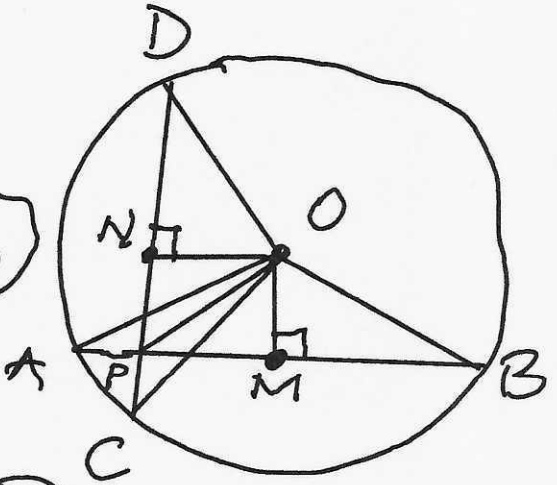


3 $\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OM}$ (2p)

$\overrightarrow{OC} + \overrightarrow{OD} = 2\overrightarrow{ON}$ (2p)

OMPN dr. $\Rightarrow \overrightarrow{OM} + \overrightarrow{ON} = \overrightarrow{OP}$ (2p)

Finalizare (1p)



4 $n^2(n^2+9)(n^2+16)(n^2+25)$ pp (1p)

$(n^4+25n^2)(n^4+25n^2+144)$ (2p) $t = n^4 + 25n^2$

$t(t+144) < (t+72)^2 \Rightarrow t(t+144) \leq (t+71)^2$ (1p)

$\Rightarrow t \leq 2520$ (1p) $\Rightarrow n \leq 6$ (1p)

$n = 0$ sol. unică (1p)

CLASA A X-A
BAREM

(1) $\overline{z_1+z_2+z_3}=0$ (4p) $\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}=0$ (2p)

$z_1^2+z_2^2+z_3^2=(z_1+z_2+z_3)^2-2z_1z_2z_3\left(\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)=0$ (2p)

Cazul $|z_1|=\dots=0$ (1p)

(2) $A=[\lg 1]+\dots+[\lg 9]$

$+[\lg 10]+\dots+[\lg 99]$ (4p)

$=9\times 0+90\times 1=90$ (3p)

(3) $\left(x^{1+\frac{1}{2}}\right)^{\frac{1}{4}}-\frac{\left(\left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}}+1\right)^{\frac{1}{7}}}{\left(x^{1+\frac{1}{2}}\right)^{\frac{1}{12}}}$ (2p)

$=x^{\frac{3}{8}}-\frac{x^{\frac{1}{4}}}{x^{\frac{1}{8}}}$ (2p) $=x^{\frac{3}{8}}-x^{\frac{1}{8}}$ (1p)

$t=x^{1/8}\Rightarrow t^3-t=6$ (1p) $\Rightarrow t=2, x=256$ (1p)

(4) f nu e inj (3p); f nu e surj (4p) astfel:

a) $f(1)=0$ (1p); $f(4)=0$ (2p)

b) Pp. că există $n\in\mathbb{N}^*$ a.î. $f(n)=\frac{1}{3}$.

$k-\log_2 n-\log_4 n=\frac{1}{3}$, $k=[\log_2 n]+[\log_4 n]$ (1p)

$k-\frac{3}{2}\log_2 n=\frac{1}{3}$ (1p) $\log_2 n=\frac{2}{3}\left(k-\frac{1}{3}\right)$ (1p)

$2^{\frac{2}{3}\left(k-\frac{1}{3}\right)}=n\Rightarrow 2^{2(3k-1)}=n^9$ (F), unul nu e CUB (1p)

Obs. Nu există nicio funcție surjectivă $f:\mathbb{N}^*\to[0,2)$, deoarece $\mathbb{N}^*$ este numărabilă și $[0,2)$ este de puterea continuului.

CLASA A XI A

BAREM

(1) $A^3 = -I_2$ (3p) $A^{2025} = (A^3)^{675}$ (2p)

$= (-I_2)^{675}$ (1p) $= -I_2$ (1p)

(2) a) $\lim\limits_{n\to\infty} \sqrt[n]{(2n)!} = \lim\limits_{n\to\infty} \frac{(2n+2)!}{2n!}$ (2p)

$= \lim\limits_{n\to\infty} (2n+1)(2n+2) = \infty$ (1p)

b) $1 < \sqrt[n^3]{(2n)!} < \sqrt[n^3]{(2n)\cdot(2n)\cdots(2n)}$ (1p)

$= (2n)^{\frac{2}{n^2}} < (2n)^{\frac{2}{n}}$ (1p) $\longrightarrow 1$ (2p)

$\Rightarrow \lim\limits_{n\to\infty} \sqrt[n^3]{(2n)!} = 1.$

(3) $f(x) \le f\left(\frac{x}{2^n}\right)$ și $f(x) \ge f\left(\frac{x}{3^n}\right)$ (2p)

pentru $n \to \infty$: $f(x) \le l$ și $f(x) \ge l$ (2p)

$f(x) = l$, $x \neq 0$ (2p) $f(0) = \alpha$ oarecare (1p)

(4) $a_n < 3$ (inducție) (1p)

$a_{2n} = \sqrt{6 + a_{2n-1}} = \sqrt{6 + \sqrt[3]{24 + a_{2n-2}}}$ (1p)

$a_0 < a_2 \Rightarrow (a_{2n})$ crescător (ind.) (1p)

$a_{2n+1} = \sqrt[3]{24 + a_{2n}} = \sqrt[3]{24 + \sqrt{6 + a_{2n-1}}}$ (1p)

$a_1 < a_3 \Rightarrow (a_{2n+1})$ crescător (ind.) (1p)

$a_{2n} \to 3$; $a_{2n+1} \to 3 \Rightarrow a_n \to 3$ (2p)

CLASA A XII-A
BAREM

(1) $x*y = x(y^3)^5[(y^{-1})^2]^7$ (2p) $= (y^3)^5 x[(y^{-1})^2]^7$ (2p)

$= (y^3)^5 \cdot [(y^{-1})^2]^7 * x$ (2p) $= y*x$. (1p)

(2) $G = \operatorname{Im} f \Rightarrow G = (1, \infty)$ (2p)

$f(xy) = f(x) \circ f(y)$ (2p)

$2xy+1 = (2x+1) \circ (2y+1)$ (1p) $\quad 2x+1 = u$, $2y+1 = v$ (1p)

$u \circ v = 2 \cdot \frac{u-1}{2} \cdot \frac{v-1}{2} + 1$ (1p)

(3) $t = \frac{\pi}{2} - x \Rightarrow I = \int_0^{\pi/2} \frac{\sin t + \sin^2 t}{1 + \cos t + \sin t}\, dt$ (2p)

$2I = \int_0^{\pi/2} \frac{(\cos x + \cos^2 x) + (\sin x + \sin^2 x)}{1 + \sin x + \cos x}\, dx$ (2p)

$2I = \int_0^{\pi/2} dx$ (2p) $\quad I = \pi/4$ (1p)

(4) $\int_a^b f(x)\,dx + \int_{f(a)}^{f(b)} f^{-1}(x)\,dx = bf(b) - af(a)$

(fără demo) (2p)

$f(x) = \sqrt{x^2+1} \Rightarrow \underbrace{\int_0^1 \sqrt{x^2+1}\,dx}_{I} + \underbrace{\int_1^{\sqrt{2}} \sqrt{x^2-1}\,dx}_{J} = \sqrt{2}$ (2p)

$\sqrt{I \cdot J} < \frac{I+J}{2} = \frac{\sqrt{2}}{2}$ (2p) $\Rightarrow I \cdot J < \frac{1}{2}$ (1p)